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ON THE QUANTIFICATION OF ROBUSTNESS OF STRUCTURES

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ABSTRACT

The paper first reviews different interpretations of robustness. On this basis objectives facilitating the quantification of robustness of engineering systems are formulated. Thereafter a generic framework for risk assessments of engineering systems is presented in which robustness is related to the ability of the system to sustain damages. This framework is then applied to quantify robustness of structural systems and to develop a robustness index facilitating a consistent ranking of structures according to their robustness. The proposed approach to the assessment of robustness principally takes into account the effect of redundancy, ductility, damage and failure consequences as well as strategies for condition control and intervention during the service life of structures. Finally, a simple example illustrates the use of the framework for the assessment of the robustness of a jacket steel structure subject to fatigue damage. The example shows that presently used indicators for the robustness of jacket type steel structures such as the *RIF* only capture part of the picture and illustrates the merits of a risk based framework for robustness assessments.

INTRODUCTION

Structural reliability is a subject of continuing intense discussion for the structural engineering profession. Over the last half century developments in the field of structural reliability have been substantial and as a result of this most codes for the design and assessment of structures take basis in quantitative requirements to structural reliability.

Typical structural reliability requirements are provided in terms of maximum acceptable annual failure probabilities in dependency of the consequences associated with structural failure and sometimes also in dependency of the relative costs associated with improvement of reliability. Based on such

requirements it is possible to identify reliability requirements for both structural components and structural systems.

Normally design codes take basis in a design philosophy where the individual components and also sometimes, but less frequently, the structural systems are assessed and designed considering their load carrying capacity subject to different relevant load scenarios. In the definition of the different load scenarios the different relevant types of loads are in turn considered as being the leading load and their extreme effect is combined with the corresponding effects of other relevant loads. Structural designs in this way explicitly take into account the relevant load scenarios including environmental extreme loads, accidental loads, earthquake loads and the effect of degradation.

When the ability of structures to sustain damages is considered the codes and existing design practices are much less specific. An overview of code provisions may be found in Ellingwood (2002). Typically requirements that structures must be robust in regard to damages, fail to be more specific than “the consequences associated with damages shall not be disproportional to the effect causing the damages”. Even though the information contained in such a statement may be substantial it is highly ambiguous. In effect the engineers and the owners of structures have little help on the quantification of robustness and no clear definition on acceptability of robustness. Several researchers have been considering the problem of quantifying robustness, however, so far most investigations address general aspects of systems reliability or consider special configurations of structures, see e.g. Ellingwood and Leyendecker (1978), Feng and Moses (1986) and Rausand and Hojland (2004). More general and practical applicable directives for ensuring the robust performance of structures are urgently needed.

The present paper sets out with an overview of different interpretations of robustness from different fields of engineering and research and basic steps in the assessment of robustness are formulated. Subsequently a new generic risk assessment framework frame work is presented in accordance with recent developments within the Joint Committee on Structural Safety (JCSS, 2001). Taking basis in the generic risk assessment framework and recent works in Maes et al. (2005), Baker et al. (2005) and Baker et al. (2006) it is shown how robustness assessments of two principally different types of systems may be performed, namely systems which may not easily be discretized into components and sub-systems such as eco-systems and systems such as steel jacket structures which may. For structural systems it is described how the robustness may be quantified and how the robustness of different structural systems may be compared. The suggested approach assesses robustness in the context of decision making such that not only the performance of damaged structures are considered in regard to various relevant loading conditions but also the effect of human interventions, monitoring schemes and inspection and maintenance strategies are taken into account. The robustness of principal structural systems is quantified and indicators of robustness identified. Indexes of robustness are suggested which allow for a consistent ranking of different structural concepts. Finally it is shown how the suggested framework may be utilized in the field of offshore engineering for the quantification of the robustness of jacket type steel structures and it is discussed how the commonly applied indicators of system characteristics such as the Reserve Strength Ratio (RSR) and the Residual Influence Factor (RIF) correlate with robustness.

INTERPRETATIONS OF ROBUSTNESS

The definitions of robustness vary greatly, as seen in Table 1, which includes definitions from engineering as well as similar concepts from quality control, biology, statistics, control theory, linguistics, etc. This Table is based on Baker et al. (2005) and Maes et al. (2005).

The ideas in Table 1 touch on essentially every aspect of what is instinctively found interesting about robustness. Robustness is related to the acceptable behavior of certain performance characteristics or properties of a system that are difficult to quantify or to parameterize (i.e. to describe the dependence using quantitative variables); and with which it is therefore difficult to associate a metric or a norm. Robustness is also a measure of the sensitivity of certain qualitative features in a system in regard to changes in system composition, system state, fundamental assumptions regarding the system and generally unexpected systemic disturbances.

It should be noted that robustness is defined for specified performance characteristics of a given system, with specified perturbations being applied to the system. It makes no sense to speak of a system being robust without first specifying both the desired system performance characteristics and the perturbations of interest. Robustness is related to the degree to which the system performance characteristics are affected by the specified perturbations.

Overall, there appear to be clear and identifiable common themes in the examples considered above which lead us to

conclude that a robustness assessment must involve the following steps also shown in Figure 1:

1. A system must be identified and clearly defined.
2. Specific system objectives must be identified: system robustness relates to certain desirable system objectives (performance characteristics or properties).
3. Specific perturbations such as the effect of hazards, internal or external influences, abnormal, deliberate or unexpected circumstances, or any other deviation from design assumptions must be identified.
4. Robustness analysis: this analysis focuses on the overall effect (consequences) of the specific perturbations (Step 3) as they affect the system objectives (Step 2).
5. Robustness index: any measures or indicators of robustness used to rank system robustness must be such that they assign high “marks” to systems which are insensitive to the specific disturbances.

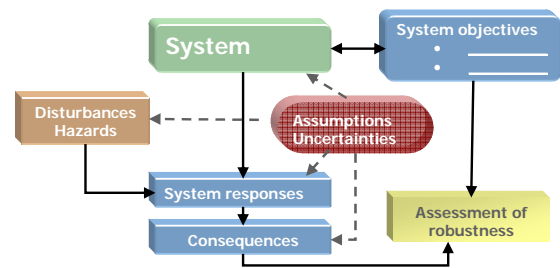


Figure 1. Schematic of the process of assessing robustness (Maes et al., 2005).

The above process can be applied to any system (and its identified features) and to specific disturbances when none of these are subject to uncertainty. In many of the examples considered above (Table 1), the robustness analyses are in fact entirely deterministic. However, in many engineering applications, the system, the system response, the cause-effect relationships, the hazards and the consequences are usually subject to considerable uncertainty. Therefore it is necessary to consider an additional element in the vulnerability assessment:

6. Risk: the assessment of robustness must account for all uncertainties associated with system assumptions (Step 1), system objectives (Step 2), the occurrence of disturbances or hazards (Step 3) and model uncertainties involved in the system consequence analysis (Step 4).

Table 1. A selection of performance characteristics associated with robustness.

| | |
|----------------------|---|
| Structural Standards | The consequences of structural failure are not disproportional to the effect causing the failure CEN (1994). |
| Software Engineering | The ability...to react appropriately to abnormal circumstances (i.e., circumstances “outside of specifications”). A system may be correct without being robust, Meyer (1997). |

| | |
|----------------------------|---|
| Product Development and QC | The measure of the capacity of a production process to remain unaffected by small but deliberate variations of internal parameters so as to provide an indication of the reliability during normal use. |
| Ecosystems | The ability of a system to maintain function even with changes in internal structure or external environment, Callaway et al. (2000). |
| Control Theory | The degree to which a system is insensitive to effects that are not considered in the design, Slotine and Li (1991). |
| Statistics | A robust statistical technique is insensitive against small deviations in the assumptions, Huber (1996). |
| Design Optimization | A robust solution in an optimization problem is one that has the best performance under its worst case (max-min rule), Kouvelis and Yu (1997). |
| Bayesian Decision Making | By introducing a wide class of priors and loss functions, the elements of subjectivity and sensitivity to a narrow class of choices, are both reduced, Insua and Ruggeri (2000). |
| Language Language | The robustness of language...is a measure of the ability of human speakers to communicate despite incomplete information, ambiguity, and the constant element of surprise, Briscoe (1997). |

the extent possible for collecting information about the constituents. This facilitates that the performance of the system may be updated through knowledge about the state of the individual constituents of the system.

Knowledge about the considered decision context is a main success factor for optimal decision making. In real world decision making lack of knowledge (or uncertainty) characterizes the normal situation and it is thus necessary to be able to represent and deal with this uncertainty in a consistent manner. The Bayesian statistics provides a basis for the consistent representation of uncertainty independent of their source and readily facilitates for the joint consideration of purely subjectively assessed uncertainties, analytically assessed uncertainties and evidence as obtained through observations.

In the context of decision making with time horizons reaching well beyond individual projects or the duration of individual decision makers the uncertainty related to system assumptions are of tremendous importance. Rather different assumptions can be postulated in regard to future climatic changes, economical developments, long term effects of pollution etc. It is obvious that if the wrong assumptions are made then also the wrong decisions will be reached.

In the process of risk based decision making where due to lack of knowledge different system representations could be valid it is essential to take this into account. Robust decisions may be identified which subject to the possible existence of several different systems will yield the maximum utility or benefit in accordance with the preferences represented by the decision maker.

Uncertainty in regard to the performance of a given system or what concerns the existence of one or another system is a major influencing factor for the decision making and it is necessary to take these uncertainties consistently into account in the process of decision making, see e.g. Faber and Maes (2005).

There exist a large number of propositions for the characterization of different types of uncertainties. It has become standard to differentiate between uncertainties due to inherent natural variability, model uncertainties and statistical uncertainties. Whereas the first mentioned type of uncertainty is often denoted aleatory (or Type 1) uncertainty, the two latter are referred to as epistemic (or Type 2) uncertainties. However this differentiation is introduced for the purpose of setting focus on how uncertainty may be reduced rather than calling for a differentiated treatment in the decision analysis. In reality the differentiation into aleatory uncertainties and epistemic uncertainties is subject to a defined model of the considered system. The relative contribution of the two components of uncertainty depends on the spatial and temporal scale applied in the model. For the decision analysis the differentiation is irrelevant; a formal decision analysis necessitates that all uncertainties are considered and treated in the same manner (Faber, 2004).

The risk assessment of a given system is facilitated by considering the generic representation illustrated in Figure 2.

GENERIC RISK ASSESSMENT FRAMEWORK

In accordance with JCSS (2001) a “system” may in general terms be considered to consist of a spatial and temporal representation of all constituents required to describe the interrelations between relevant exposures (hazards) and their consequences. Direct consequences are related to damages on the individual constituents of the system whereas indirect consequences are understood as any consequences beyond the direct consequences.

System representation

A system representation can be formulated in terms of logically interrelated constituents at various levels of detail or scale in time and space. Constituents may be physical components, procedural processes and human activities. The appropriate level of detail or scale depends on the physical or procedural characteristics or any other logical entity of the considered problem as well as the spatial and temporal characteristics of consequences. The important issue when a system model is developed is that it facilitates a risk assessment and a risk ranking of decision alternatives which is consistent with available knowledge about the system and which facilitates that risks may be updated according to knowledge which may be available at future times. Furthermore, the system representation should incorporate options for responsive decision making in the future in dependence of knowledge available then.

It is important that the chosen level of detail is sufficient to facilitate a logical description of events and scenarios of events related to the constituents of the system which individually and/or in combination may lead to consequences. In addition to this the representation of the system should accommodate to

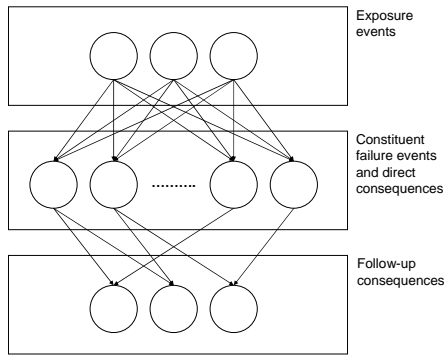


Figure 2. Generic system representation in risk assessments.

Consequence modeling

The exposure to the system is represented as different exposure events acting on the constituents of the system. The constituents of the system can be considered as the systems first defense in regard to the exposures. The damages of the system caused by failures of the constituents are considered to be associated with direct consequences. Direct consequences may comprise different attributes of the system such as monetary losses, loss of lives, damages to the qualities of the environment or just changed characteristics of the constituents. Based on the combination of events of constituent failures and the corresponding consequences follow-up consequences may occur. Follow-up consequences could be caused by e.g. the sum of monetary losses associated with the constituent failures and the physical changes of the system as a whole caused by the combined effect of constituent failures. The follow-up consequences in systems risk assessment play a major role and the modeling of these should be given great emphasis. It should be noted that any constituent in a system can be modeled as a system itself. A system could be a field of platforms with constituents being the individual platforms. The platforms in turn could also be systems with structural members as constituents. Depending on the level of detail in the risk assessment, i.e. the system definition, the exposure, constituents and consequences would be different.

In Figure 3 the system which is considered subject to a risk assessment is assumed to be exposed to hazardous events (exposures EX) with probabilistic characterization $p(EX)$. The probability of direct consequences $c_D(C_{ij})$ associated with the j^{th} state C_{ij} of the i^{th} constituent of the system due to the exposure on the constituent is described by $p(C_{ij}|EX)$ and the associated conditional risk is $p(C_{ij}|EX)c_D(C_{ij})$. The summation and integration of the conditional risk over all system constituents and states, respectively is denoted the vulnerability of the system in regard to the considered exposure. The risk due to direct consequences is assessed through the expected value of the system vulnerability over all possible exposure events.

Finally the probability of indirect consequences $c_{ID}(S_k, c_D(C))$ associated with the system state S_k due to the

exposure EX , the state of the constituents C and the associated direct consequences $c_D(C)$ is described by $p(S_k|C, EX)$ and the corresponding conditional risk is $p(S_k|C, EX)c_{ID}(S_k, c_D(C))$. The integration of the conditional indirect risk over all possible system states can be seen as a measure of robustness; indicating the ability of the system to limit the total consequences to the direct consequences for given constituent state and exposure. The risk due to indirect consequences is assessed through the expected value of the indirect consequences in regard to all possible exposures and constituent states.

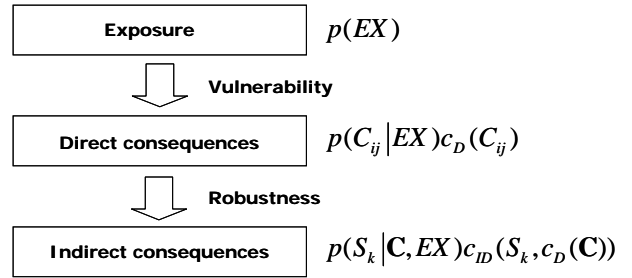


Figure 3. Suggested generic and indicator based risk assessment framework.

It should be realized that the suggested risk assessment framework is applicable at any level of scale for the assessment of a given system. It may be applied to components, sub-systems and the system as a whole; thereby the framework also facilitates a hierarchical approach to risk assessment. The definition of the system in this context becomes of tremendous significance in the definition of exposure, vulnerability and robustness. The risk assessment framework allows for utilization of any type of quantifiable indicators in regard to the exposure, vulnerability and robustness of the considered system. Due to the hierarchical structure of the risk assessment, in terms of conditional events the framework is greatly supported by modern risk assessment tools such as e.g. Bayesian Probabilistic Nets and Influence Diagrams, see e.g. Jensen (2001).

Indicators of risk

Risk indicators may be understood as any observable or measurable characteristic of the systems or its constituents containing information about the risk. If the system representation has been performed appropriately risk indicators will in general be available for both the exposure to the system, the vulnerability of the system and the robustness of the system, see Figure 4.

In a Bayesian decision making framework such indicators play an important role. Considering the risk assessment of a load bearing structure risk indicators are e.g. any observable quantity which can be related to the loading of the structure (exposure), the strength of the components of the structure (vulnerability) and the redundancy, ductility, effectiveness of condition control and maintenance (robustness).

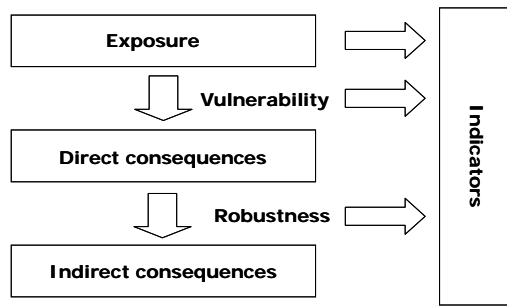


Figure 4. Risk indicators at different levels of the system representation.

ASSESSMENT OF STRUCTURAL ROBUSTNESS

From the previous sections it is clear that the notion of robustness depends critically on the system definition and the consequences. In the following, these aspects are considered for the application of the framework to structural systems for which the objectives are one of the following:

1. Contain severe system consequences.
2. Contain indirect (system) consequences in relation to direct (component) consequences.

These objectives lead to a robustness assessment framework that is particularly valuable in practice.

Structural system and consequences

The definition of the structural system is extremely critical to the analysis. It must be “wide” enough to accommodate consequences that affect the robustness objectives. Assumptions regarding the system (Faber and Maes, 2005) must be clearly considered so that uncertainties can be included during the robustness analysis. This is valid for simple assumptions such as the strength assumed for the overturning capacity of a jacket structure as well as for more long-ranging assumptions such as neglecting the effect of global warming in a coastal structures design

Exposure to hazards or disturbances

It is very clear from the above examples that it is not possible to speak of robust systems without referring specifically which hazards or disturbances the system is considered to be exposed to. For instance robustness conditional on the occurrence of an extreme wave can be investigated. Or the same investigation can be performed probabilistically, for a random extreme wave occurring at a specific location. But robustness cannot be assessed without specifying which kind of hazards are of concern.

Sensitivity of the system objectives

Robustness thus focuses on the system objectives in the face of exposure to perturbations. The following sections explore analysis frameworks that can quantify this so as to establish a basis for consistent ranking of robustness.

Robustness assessment of continuous systems

In accordance with Maes et al. (2005) the following considers robustness indicators suitable to deal with the

objective of “containing the consequences of failure C_F ” in a continuous system subject to an external hazard X .

The curves shown in the left part of Figure 5 are sample functions of failure consequences as they are conditional on the basic random variables governing the response of the system to the disturbance and they are obtained by structural or system response analysis. Sample response (a) shows an ideal situation in terms of robustness: after failure occurs the system remains relatively stable and sustains only limited or local damage. Sample response (b) shows a system that is robust to the extent that consequences of failure occur progressively as the external hazard becomes gradually more severe. Sample response (c) shows a situation where the system becomes critically harmed as soon as first damage occurs.

When integration is performed over the pdf of the hazard X itself, then the probability of exceedance of failure consequences given the occurrence of an extraordinary hazard is obtained. When this decreasing probability is plotted on a logarithmic scale a good idea is achieved of the tail behavior of the failure consequence distribution, as shown in the right part of Figure 5. The pattern obtained is similar to the sample curves obtained left. In order of decreasing robustness, (a) corresponds to fully contained consequences, (b) proportional (gradual) system consequences, and (c) cascading consequences.

As a measure of robustness, the inverse of the tail heaviness H of the log-exceedance curves in Figure 5 (right) may be considered which is easy to compute for the tail of any given probability distribution, Maes (1995): H is less than one for case (a), one for an exponential relationship between C_F and its exceedance probability (b), and large for case (c). Alternatively one could consider an equivalent measure of robustness based on the evolution of the “expected follow-up system consequence ($EFUSC$)” as a function of c :

$$EFUSC(c) = E(C_F - c | C_F > c) \quad (1)$$

This curve expresses what the likely “additional” systemic consequences will be if a disturbance has already resulted in a given level of consequences c . Both approaches result in the same robustness ranking. A detailed numerical example is given in Maes et al. (2005).

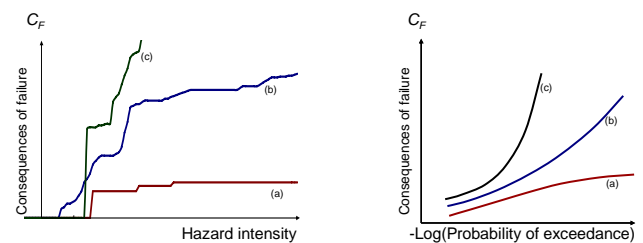


Figure 5. (left) Sample functions of failure consequences versus hazard intensity; (right) Conditional probability of exceeding failure consequences.

Robustness assessment of discrete systems

Several works address the robustness of structural systems. Lind proposed a generic measure of system damage tolerance, based on the increase in failure probability resulting from the occurrence of damage Lind (1995, 1996). Ellingwood (2005) suggests that probabilistic risk assessment can be used to assess robustness in a general manner.

In the following the approach suggested for the assessment of robustness of structural systems in Baker et al. (2005) and Schubert et al. (2005) is outlined. In Figure 6, events that may damage a system are modeled as follows. First, an exposure (E_{BD}) occurs which has the potential of damaging components in the system. If no damage occurs \bar{D} , then the analysis is finished. If damage occurs, a variety of damage states D can result. For each of these states, there is a probability that system failure F results. Consequences are associated with each of the possible damage and failure scenarios. The event tree representation in Figure 6 is a graphical tool for evaluating event scenarios that could occur to the system, and it also incorporates the associated probabilities of occurrence.

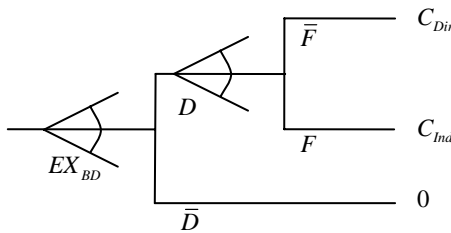


Figure 6. An event tree for robustness quantification, Baker et al. (2005).

The symbols used in Figure 6 are defined as follows:

| | |
|-----------|---|
| EX_{BD} | Exposure before damage |
| D | Component Damage (refers to no damage) |
| F | System failure, or “failure” (refers to no failure) |
| C_{Dir} | Direct consequences (related to component damage) |
| C_{Ind} | Indirect consequences (related to system failure) |

An exposure is considered to be any event with the potential to cause damage to the system; damage could come from extreme values of design loads such as snow loads, unusual loads such as explosions, deterioration of the system through environmental processes such as corrosion, errors or other disturbances. Damage refers to reduced performance of the system components, and system failure refers to loss of functionality of the entire system. In the case that a design allows for some degree of reduced function (e.g., an allowance for some corrosion), then damage should refer to reduced function beyond the design level.

Structural design is traditionally based on member design where the reliability of each individual structural member is ensured at a level which is acceptable in accordance with the (direct) consequences associated with failure of the member, JCSS (2001). The structural systems aspects are not directly accounted in this way. In Figure 6, however, they are taken into

account in terms of the indirect consequences, i.e. those related to the effect of the member failures.

With the event tree defined in Figure 6, it is possible to compute the system risk due to each possible event scenario. This is done by multiplying the consequence of each scenario by its probability of occurrence, and then integrating over all of the random variables in the event tree. Following Baker et al. (2005) the risk corresponding to each branch is:

$$R_{Dir} = \int \int C_{Dir} P(\bar{F} | D = y) P(D = y | EX_{BD} = x) \times P(EX_{BD} = x) dy dx \quad (2)$$

$$R_{Indir} = \int \int C_{Indir} P(F | D = y) \times P(D = y | EX_{BD} = x) P(EX_{BD} = x) dy dx \quad (3)$$

In order to now quantify robustness, consider that a robust system is considered to be one where indirect risks do not contribute significantly to the total risk of the system. With this in mind, the following index of robustness (denoted I_R) is proposed, which measures the fraction of total system risk resulting from direct consequences

$$I_R = \frac{R_{Dir}}{R_{Dir} + R_{Ind}} \quad (4)$$

The index takes values between zero and one depending upon the source of risk. If the system is completely robust and there is no risk due to indirect consequences, then $I_R = 1$. At the other extreme, if all risk is due to indirect consequences, then $I_R = 0$.

In Schubert et al. (2005) the presented framework is investigated in some detail for general series and parallel systems. However, by examining Figure 6 and the above equations, several qualitative trends between system properties and the robustness index can be identified.

First, this index measures only relative risk due to indirect consequences. The total system risk should be deemed acceptable through other criteria prior to robustness being considered. A system might be deemed robust if its direct risk is extremely large (and thus large relative to its indirect risk), but that system should be rejected on the basis of reliability criteria rather than robustness criteria. Guidelines for evaluating acceptable reliability can be found in existing codes (e.g. JCSS (2001)).

Second, the index depends not just upon failure probabilities of damaged systems, but also upon the relative probabilities of the various damage states occurring. Thus, a structure could be designed to have a low failure probability after an individual column is removed, but if it is deemed more likely that an exposure would cause the loss of two columns and if the structure as a structural system is not reliable in this situation, then it could still be deemed non-robust.

Third, the index accounts for both the probability of failure of the damaged system and the consequences of that failure. For instance, if sensing systems were able to detect damage and signal an evacuation before failure could occur, then robustness could be increased without changing the probabilities of

damage or failure. Thus, the possibility of detection and the time between damage and failure can be accounted for in an appropriate manner. The property of robustness depends upon system properties such as redundancy, ductility, load redistribution and damage detection, but it also depends upon failure consequences. This ability to incorporate consequences as well as probabilities is an important new development.

Fourth, this index can be easily extended to account for multiple exposures, or more complicated event trees than the one in Figure 6. The robustness index will still be equal to the sum of direct risk divided by the sum of total risk.

Fifth, by other important aspects of system performance, the framework can be used for decision-making regarding design actions, including maintenance, inspection, monitoring and disaster preparedness. This is illustrated in Figure 7 where the additional symbols are defined as:

- a_d Design actions, including maintenance, inspection, monitoring and disaster preparedness
- I Indication of damage, which triggers a response action (refers to no indication)
- a_r Response actions
- EX_{AD} Exposure after damage

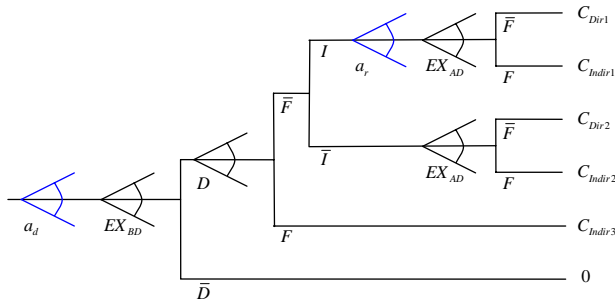


Figure 7. An event tree that incorporates system choice and post-damage exposures, Baker et al. (2005).

By incorporating post-damage exposures, the framework can now account for the increased vulnerability of the structure in the future. Further, the opportunity to intervene through response actions (a_r) is now modeled explicitly. These actions are conditional on the indication of a damage (the probability of which is affected by the inspections and monitoring actions which are here assumed to be part of the design decisions). Based on the damage level of the system, and the actions taken as a result of detection, the system has a probability of failure due to post-damage exposures (EX_{AD}).

It is implied that if damage is indicated, then action will be taken either to reduce failure consequences (e.g., by evacuating a structure) or the probability of failure (e.g., through repairs). The choice of post-detection action is part of the definition of the system. The probability of damage detection will be dependent upon actions to inspect the system, and on the type of damage and type of exposure causing damage. For example, damage from explosions will likely be detected, while corrosion of an inaccessible component may not be detected.

The basic choice of design action (a_d) is now also explicitly included at the beginning of the tree. Actions include design of the physical structure, maintenance to prevent structural degradation, inspection and monitoring for identifying damages, and disaster preparedness actions. These actions, along with the post-damage response actions, are included here because will affect the probabilities and consequences associated with the other branches, and so this decision tree can be used as a tool to identify actions which minimize risk and maximize robustness in a system. When alternative systems have varying costs, then these costs should be included in the consequences (and the branch of the tree corresponding to will no longer have zero consequences for some system choices). With this formulation, a pre-posterior analysis can be used to identify systems which minimize total risk.

For a given set of actions, the risks associated with each branch can be computed as before. For example, the indirect risk R_{Ind_2} would now be computed as (Baker et al. (2005):

$$R_{Ind_2} = \int \int \int C_{Ind_2} P(F | D = y, \bar{I}, EX_{AD} = z) \times P(EX_{AD} = z | D = y, \bar{I}) P(\bar{I} | D = y) P(\bar{F} | D = y) \times P(D = y | EX_{BD} = x) P(EX_{BD} = x) dz dy dx \quad (5)$$

The corresponding index of robustness can be calculated using a direct generalization of Equation (4):

$$I_R = \frac{\sum_i R_{Dir_i}}{\sum_i R_{Dir_i} + \sum_j R_{Ind_j}} \quad (6)$$

EXAMPLE

As an illustration of the suggested approach for the assessment and quantification of robustness a jacket steel platform is considered. It is assumed that the platform is being designed and an assessment of the robustness of the platform is desired. In principle an overall robustness assessment could be performed by considering all possible exposures including e.g. accidental loads, operational errors and marine growth. However, for the purpose of illustration the following example only considers the robustness of the platform in regard to damages due to fatigue failure of one of the joints in the structure. The scenario considered here is thus the possible development of a failed joint due to fatigue crack growth and subsequent failure of the platform due to an extreme wave. By examination of Figure 6 and Equation (4) it is realized that when only one type of damage exposure is considered and only one joint is considered the robustness index does not depend on the probability of the exposure and also not on the probability of damage. In general when all potential joints in a structure are taken into account and when all possible damage inducing exposures are considered a probabilistic description of exposures and damages would be required as indicated in Equations (5)-(6).

The further assessment of the robustness index thus only depends on the conditional probability of collapse failure given

fatigue failure as well as the consequences of fatigue damage and collapse failure. To this end the concept of the Residual Influence Factor (*RIF*) is applied. Based on the Reserve Strength Ratio *RSR* (Faber et al., 2005) the *RIF* value corresponding to fatigue failure of joint *i* is given as:

$$RIF_i = \frac{RSR_i}{RSR_0} = \frac{RSR \text{ based on joint } i \text{ failed}}{RSR \text{ based on no members failed}} \quad (7)$$

For illustrational purposes collapse failure is modeled by the simple limit state function:

$$g(x) = R - bH^2 \quad (8)$$

where it is assumed that the resistance *R* is Log-normal distributed with a coefficient of variation equal to 0.1, the bias parameter on the load *b* is Log-normal distributed with a coefficient of variation equal to 0.1 and the wave load *H*² is assumed Gumbel distributed with a coefficient of variation equal to 0.2. Defining the *RSR* through the ratio:

$$RSR = \frac{R_c}{b_c H_c^2} \quad (9)$$

In Equation (9) the indexes *C* refer to characteristic values. These are defined as 5%, 50% and 98% quantile values for *R*, *b* and *H*², respectively and are calculated from their probability distributions. Using Equations (7)-(9) it is directly possible to relate the *RSR* and the *RIF* factors to an annual probability of collapse failure of the platform. Assuming that the *RSR* for the considered platform is equal to 2, the annual probability of failure given fatigue failure is shown as function of the *RIF* in Figure 8.

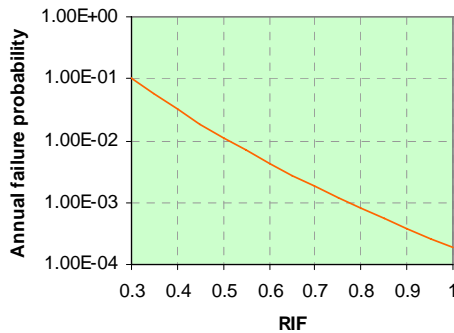


Figure 8. Relationship between the annual probability of collapse failure and the *RIF* for *RSR*=2.0.

It is now straightforward to calculate the robustness index *I_R* as defined in Equation (4) by consideration of Figure 4. In Figure 9 the robustness index *I_R* is illustrated as a function of the *RIF* and the ratio between the costs of collapse failure and the costs of fatigue failure of one joint, i.e. C_{Ind} / C_{Dir} .

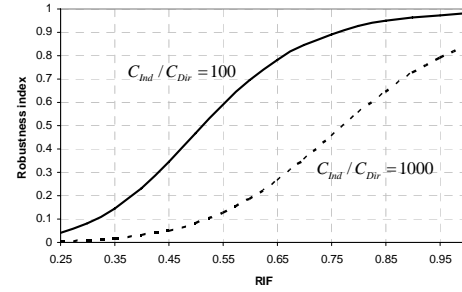


Figure 9. Relationship between the robustness index and the *RIF* factor for different relations between damage and collapse costs.

It is seen from Figure 9 that the robustness of the structure in regard to fatigue damages correlates well with the *RIF* value, however, the strength of the *RIF* value as an indicator of robustness depends strongly on the consequences of damage and failure. For the present example the case where $C_{Ind} / C_{Dir} = 1000$ might be the most relevant in which case the robustness is the lowest. From this observation it becomes clear that consequences effectively play an important role in robustness assessments and this emphasizes the merits of risk based approaches. As mentioned earlier and illustrated in Figure 7 the robustness may be improved by implementation of inspection and maintenance. Thereby the probability of fatigue failures as well as structural collapse may be reduced at the costs of inspections and possible repairs.

CONCLUSIONS

The present paper addresses the problem of quantification of robustness taking basis in a generic framework for risk assessment. Robustness is specifically considered for systems depending on whether they may be decomposed in to components and subsystems but the idea in both cases is to quantify robustness through the degree that consequences are contained by the systems. For structural systems which may easily be represented in terms of interrelated components robustness is quantified through the direct risk, i.e. the risks of component failure and the indirect risk, i.e. the risk of system failure given component failure. An index of robustness is formulated which consistently ranks the robustness of different decisions of design, inspection and maintenance and emergency preparedness. The presented concept is new in the sense that it combines the effects of redundancy, ductility, consequences of failure and decision making in regard to design and intervention during use by means of risks. Thereby it facilitates a consistent assessment of robustness and thus provides a helpful supplement to traditional member based design.

On a simplified example related to offshore engineering it is illustrated how the assessment of robustness in regard to fatigue damages and sub-sequent structural collapse for jacket type steel structures may be performed. This study clearly show that the strength of presently applied indicators for robustness such as the *RIF* factor strongly depend on the consequences of

damage and collapse and emphasizes the relevance of a risk based framework for the assessment of robustness.

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